**Tax Reform in a Heterogeneous Agent Model**

1. Model Overview

This study examines an economy composed of many individual households that make decisions over time about their consumption and savings. Households accumulate assets as a way to save, and their decisions must be made without certainty about future labor productivity—some individuals have higher or lower productivity, and these levels shift randomly from one period to the next.

Each household’s objectives are twofold: (1) gaining satisfaction from consumption in every period, and (2) making sure enough resources are set aside for future consumption stability, given that there is uncertainty about future productivity. Because the future is discounted (households prefer present consumption over the same amount consumed later), they still need to maintain some savings to buffer productivity downturns.

Taxation adds another layer of complexity: the government requires revenue and might collect taxes in ways that are more or less progressive. A flat tax imposes the same rate on every household, while a progressive tax increases the tax rate for higher incomes. Our goal is to understand how shifting from a flat tax to a more progressive one—while keeping other factors constant—affects variables such as total savings, inequality, wages, and the interest rate.

A representative firm combines capital (the sum of assets households hold) with labor (the workforce) to produce output. In equilibrium, the firm’s demand for capital must align with the capital households choose to hold, and total labor demand must match the sum of all effective labor supplied by households (their productivity times labor effort).

Relevance: By allowing households to decide how much to save when income can vary significantly and taxes can be flat or progressive, we can study how real-world policies shape income distribution, total savings in the economy, and government finances.

2. Calibration Steps

2.1. Productivity Process

A key component is how each individual’s productivity evolves. We assume productivity follows a random process with some persistence (the tendency to remain at or near the current productivity level) and the possibility of upward or downward shocks. To simplify, we convert what is initially a continuous process into a limited number of discrete “states,” typically using Tauchen’s method.

Each period, a household’s productivity transitions to another state according to probabilities that reflect parameters like persistence and volatility. Adjusting these parameters ensures the model mirrors observed patterns of how people’s labor earnings change over time in reality.

2.2. Production Parameters

On the production side, we rely on typical U.S. economic data. The labor share of output (the portion of total output that goes to workers) is often between 60% and 70%. We approximate it at around 64%, leaving roughly 36% as the share of capital. Other important targets include:

* An average interest rate of about 4%.
* An investment-to-output ratio near 20%.
* A government spending share of output also around 20%.

We fine-tune parameters such as capital depreciation rates, total factor productivity, and baseline tax rates to match these targets. This ensures that model outcomes resemble a simplified version of the U.S. economy.

2.3. Solving for the Discount Factor

Households’ inclination to save is tied to their patience, captured by the discount factor. Under a flat tax (i.e., no progressivity), we fix the wage and interest rate to match established targets. We then iteratively adjust the discount factor so that the total assets households wish to hold align with the firm’s capital needs. If our guess for the discount factor is too low, households will save less than needed; if too high, they will try to save more than the economy can absorb. By recalibrating the guess until supply and demand match, we identify the equilibrium discount factor.

2.4. Solving for the Level of Taxation Under Progressivity

With the discount factor in place, we introduce more progressivity into the tax code so that higher-income households face higher marginal rates, while still aiming to collect the same share of total output (e.g., 20%) as in the flat-tax case. Because changing progressivity influences households’ behavior toward labor and savings, it can shift wages and interest rates. Hence, we conduct another search: we propose a new “tax level,” solve the model, and check whether the government’s revenue remains the desired share of output. We continue until we reach the correct overall revenue target under the new, more progressive system.

3. Numerical Solution

Value Function Iteration (VFI) is central to solving each household’s problem. The procedure is as follows:

1. Asset Grid: We establish a list (grid) of possible asset holdings a household might carry into the next period.
2. Choice Over Assets: For each productivity state and current asset level, a household decides how much to set aside for the future.
3. Optimization: The chosen asset level maximizes overall satisfaction (considering taxes on labor income, future uncertainty, and the returns on saving).
4. Iteration: We repeatedly update the “value function” (an expression of total expected well-being given any state) until it converges. This yields a policy function showing how much a household saves for each combination of productivity and assets.

Stationary Distribution: Even with individual decisions pinned down, we must find how households are spread across asset levels and productivity states for the whole economy. We start with a guess about this distribution and apply both the policy function and productivity transitions. Repeating this process eventually yields a stable (stationary) distribution that does not change over time.

Market Clearing: Summing all assets across households gives the total capital supply. This supply must match the firm’s desired capital in equilibrium if the interest rate is to remain at its model-determined level (or if we solve it endogenously). This linkage ensures that both the household and production sides are consistent with one another.

4. Results and Comparisons

After the calibration and solution, we compare two steady-state outcomes:

1. Flat-Tax Economy:  
   Here, all labor income is taxed at the same rate. Households with different productivity levels do not face substantially different marginal taxes. We pick wages and interest rates (matching data) and deduce the required discount factor for aggregate saving to meet the economy’s capital demand. We also see what uniform tax rate is needed to collect a predetermined fraction of output.
2. Progressive-Tax Economy:  
   With all other model components unchanged, the tax schedule becomes more progressive. Higher earners now pay a larger portion of their income in taxes, so we must find a new baseline tax level that still produces the same share of revenue (e.g., 20% of output). Because household saving incentives change, we allow wages and interest rates to adjust accordingly.

Key Observations:

* Interest Rate and Wage: Under the flat tax, these are somewhat set by calibration. In the progressive case, they are re-solved. When high-income households face higher taxes, they may opt to save less, thus reducing aggregate capital. This can nudge interest rates upward (and wages downward), though specific outcomes depend on parameter values.
* Tax Rate: The newly determined tax level with progressivity may not match the old flat rate precisely because the shape of the tax schedule is different. This redistributes the burden among different income groups while maintaining the same total government revenue.
* Capital-to-Output Ratio: Changing progressivity reshapes saving decisions. For instance, if high earners find saving less attractive, total capital might dip, which would alter the overall capital-to-output ratio. The exact direction of this change depends on various interactions within the model.
* Inequality Measures: We monitor how taxes affect after-tax labor income inequality and asset distribution. Progressive taxes typically narrow the gap in after-tax labor income, lowering the Gini coefficient. However, asset inequality may remain high because wealth accumulates gradually over time.

5. Plots and Economic Insights

We illustrate these findings with several visual aids:

* Value Functions: Graphs showing how a household’s overall expected well-being (lifetime utility) depends on its asset holdings under different productivity levels. Higher productivity or more assets generally leads to higher value functions due to greater consumption possibilities.
* Policy Functions: These depict the next-period asset choice given current assets and productivity. By comparing policy functions across different tax structures, we see whether more progressive taxes reduce the return to work and saving, especially for top earners.
* Asset Distributions: We can show the proportion of households owning various levels of assets, revealing whether wealth is concentrated among a small group or more evenly spread out. Shifts in the tax system can alter the shape of this distribution.
* Lorenz Curves: These curves visualize income and wealth concentration. Greater tax progressivity usually flattens the after-tax income curve, demonstrating a reduction in inequality at the higher end of the income scale.

Through these figures, we observe that increasing progressivity tends to compress after-tax income differences. However, it also triggers changes in saving behavior that can influence aggregate capital and, in turn, affect wages and interest rates.

6. Conclusion

This project highlights how the design of taxes affects both economic aggregates and distribution. Moving from a purely flat labor tax to a more progressive one, while holding government spending constant, generally lowers after-tax income inequality because top earners shoulder more of the tax load. Yet this can also influence the overall saving rate in the economy and potentially shift equilibrium prices such as the wage and the interest rate—especially if the model allows for free adjustment of those prices.

In policy discussions, there is often debate about balancing fairness and efficiency: more progressivity can reduce inequality, yet it may also alter work or saving incentives. The simplified framework here represents a slice of that bigger conversation, demonstrating that raising progressivity can make incomes more equal after taxes but might also reduce incentives to accumulate capital. Final outcomes depend on how the tax schedule is crafted, how sensitive labor supply is, and the manner in which public funds are spent.

By contrasting a flat-tax scenario with one featuring moderate progressivity, we gain a clearer picture of how tax policy shapes household behavior and overall market equilibrium. It underscores that tax policy is not merely a question of who pays what share, but also how it permeates through the broader economy, influencing saving, wages, and the distribution of resources.